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CRISIS DYNAMICS AND DETERRENCE:

A REFORMULATION OF THE CLASSIC CALCULUS

APRIL 1977

By:

LEE E. DUTTER

AND

ABRAHAM R. WAGNER

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ANALYTICAL ASSESSMENTS CORPORATION
Post Office Box 9695
Marina del Rey
California 90291

4640 Admiralty Way, Marina del Rey, California 90291 • 213/822-2571

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While the importance of timing in crises is generally acknowledged, it involves a major assumption which is seldom explored and on which little formal work has been done. Most often, crisis models and studies are static in nature, assuming the time component to be a characteristic of the crisis situation, rather than a variable in the equation.

Any attempt to model specific crises reveals the fundamental nature of the timing element, and the need to incorporate changes in utilities and

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PREFACE

It it possible to trace the development of crisis analysis and crisis management in the modern era from the major international crises which have confronted the United States, such as Pearl Harbor and the Cuban Missile Crisis, through the inquiry efforts which followed and subsequent efforts to improve our national ability to monitor international events, predict potential areas of conflict and "manage" ongoing events to deter conflict. Such analyses have, increasingly, moved from historical and journalistic accounts of prior "failures" toward more theoretically based efforts to analyze crisis situations, and automated tools which will enable key decision-makers to maximize their ability to deal with critical crisis problems.

The present effort falls into the general area of "basic" crisis research, and considers the essential element of timing in deterrence situations. Working toward the aim of providing a more realistic framework upon which to assess the actions of individual actors in crisis situations, the current technical report expands upon some traditional notions of deterrence theory. Subsequent reports in this series will present the framework as a developed dynamic crisis model, and apply the model to crisis data.

A number of individuals in the Department of Defense and its Advanced Research Projects Agency have made contributions to the present work, both before its inception as a formal project and afterward. The authors would like to express their gratitude to Drs. Robert A. Young and Steven Andriole of the Cybernetics Technology Office, Defense Advanced Research Projects Agency, Dr. Gerald Sullivan, currently on leave from DARPA with The Center for International and Strategic Studies, Georgetown University, and Prof. Patrick J. Parker, U.S. Naval Postgraduate School.

The research and preparation of this study have benefited from the aid and counsel of Dr. Raymond Tanter, Marcy G. Agmon, and Dr. Jeffrey Richelson of Analytical Assessments Corporation. While each of these individuals has made a contribution to this study, the authors bear sole responsibility for any errors of fact or judgment.

CONTENTS

SUMMARY

While the importance of the timing element is generally acknowledged as an essential part of crisis analysis, it involves a major assumption which is seldom explored and on which little work has been done to incorporate the timing element as a formal component of the model. Most often, crisis models and studies of the decision-making process are static in nature, assuming the time component to be a characteristic of the crisis situation, rather than a variable in the equation.

Any attempt to model specific crises reveals the fundamental nature of the timing element and the need to incorporate changes in utilities and other variables over the course of a crisis. Prior efforts to apply utility theory to deterrence situations have encountered five major problem areas:

- the concepts of utility theory that have been applied lack conformity with other, more advanced applications which extend the theory to dynamic situations;
- some concepts in the field of statistical decision theory that have considerable relevance to deterrence theory have not been explicitly applied, which relate to the specific problems of decision-making under conditions of risk and uncertainty;
- deterrence theory, as previously applied, fails to distinguish between levels of deterrence;
- there has been a neglect of the dynamics of the deterrence situation, in terms of decision-makers' evaluations of outcomes and factors relevant to such evaluations; and
- the empirical applicability of deterrence theory has remained complex and abstract.

The present research focuses on the first two problem areas, presenting a more direct application of utility theory to deterrence and introduces some additional concepts from statistical decision theory to more clearly delineate the statics of the deterrence situation. Subsequent reports will deal with the balance of the problem areas identified.

In reformulating the "classic" deterrence calculus, the analysis assumes

that there are two actors in a deterrence situation, and that the assessment is being made from the perspective of a potential aggressor. Additional assumptions are made about the rationality of the decision-makers, in terms of their ability to rank potential outcomes, and their ability to choose a course of action which maximizes expected utility. The analysis goes on to construct the potential aggressor's utility function over the set of possible outcomes on a [0,1] normalized basis.

Successful deterrence of a potential aggressor depends upon three fundamental factors:

- his preference ordering over the possible outcomes of the deterrence situation, if and only if
- the Status Quo is his middle ranked outcome, compared to the relative valuations of the Status Quo compared to Victory and Defeat, and
- his estimate of the probability of a Victory in an attack situation.

It is possible to make this assessment more specific and explicit through the use of statistical decision theory, in the evaluation of a decision-maker's preference ordering (I) and probability estimate Pr(V). By introducing a range of possible risk functions for the decision-maker it is possible to account for the various individual personality types which exist in the real world. The significance of this approach lies in the fact that two decision-makers, both of whom are risk averse, neutral or acceptant and faced with the same decision situation, make different choices depending on which type of probability estimates they make.

I. INTRODUCTION

One of the elements most critical to defense and national security decision-makers in crisis situations is the importance of the time frame in which the crisis develops and the decisions taken which determine its outcome.[1] Increasingly it appears to be the case that international crises take place in ever shorter time spans, with the timing elements becoming a significant factor in the decision process. As crises unfold in limited time frames, specific demands are placed on a nation's intelligence and early warning capabilities, its decision-making or national command authority, the readiness of its military force, and the command,control,and communications functions of its defense establishment.

In view of the set of problems created for a nation's policy-makers and defense leadership by crisis management within the context of limited time frames, it is important to understand the essential dynamic elements in crisis decision-making, and the impact which these elements are likely to have on those faced with the difficult crisis decisions.

1.1 Timing and Crisis Decision-Making

While the importance of the timing element is generally acknowledged as an essential part of crisis analysis, it involves a major assumption which is seldom explored and on which little work has been done to incorporate the timing element as a formal component of the model. Most often, crisis models and studies of the decision-making process are static in

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[1] Compare, for example, Israel's experience in the crisis surrounding the outbreak of the 1967 Six Day War, which occupied some two weeks in May 1967, with that surrounding the outbreak of the 1973 October War, which took place in a matter of two days. See Abraham R. Wagner, *Crisis Decision-Making: Israel's Experience in 1967 and 1973.* New York: Praeger Publishers, 1974. This problem is further analyzed in R. D. McLaurin, Mohammed Mughisuddin and Abraham R. Wagner, *Foreign Policy: Issues and Policy-Making in Four Middle Eastern States.* New York: Praeger Publishers, 1976.

nature, assuming the time frame to be a characteristic of the crisis situation, rather than a variable in the equation.

The examination of specific crises reveals the fundamental nature of the dynamic element and the crucial relationship which timing plays in the actions taken and in the type and nature of the decision reached. Looking, for example, at the decisions taken by the Israeli leadership prior to the 1967 Six Day War and the 1973 October War, it is possible to witness major differences in the decision process and the decisions taken by the nation's political and military leadership. While it is clear that in the 1973 case the time frame for the dissemination of intelligence, evaluation,and ultimate policy was far shorter than in the 1967 case, no formal framework exists for the dynamic analysis of these differences and the policy decisions reached.[2] Looking at United States and Soviet actions in various crises over the past two decades, it is possible to witness some of these same fundamental problems raised by the timing involved.

Specifically, it is possible to identify several critical areas in the policy-making process where the timing element, and a better understanding of the functional problems involved, are of major importance:

- in the demands placed on intelligence collection efforts, analysis and dissemination to key military and political decision-makers;
- in shaping the constraints on the nation's decision-makers, such as a national command authority, National Security Council, Joint Chiefs of Staff,or other group by the set of feasible options within the perceived time frame;
- in planning for essential command, control, communications (C^3),and logistic systems to deal with crisis

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[2]See, for example, Avi Shalim, "The Failure of Israeli National Intelligence Estimates," *World Politics,* Winter, 1976.

management and potential conflict situations.

For all of these elements there has already been considerable progress and technical development, based on the operating assumption that modern crisis management will require effective and efficient operation of each of these systems. The United States intelligence community, for example, has long operated on the assumption that in the intelligence process the nature of the problem must dictate which methods and approaches of analysis must be utilized. To the extent that timing has been perceived as being critical, the intelligence community has moved toward long-range data collection and analysis.

Similarly, the Department of Defense and the armed services have taken major steps to improve United States capabilities in areas such as information processing, worldwide C^3, logistics, targeting, and analysis of potential conflict scenarios. Taken together, these programs dramatically enhance the capability of the United States to react to and effectively deal with a broad range of international crises.

While all of these efforts implicitly deal with the timing element, it remains for a formal analysis to integrate this factor as a principal component in crisis analysis. By undertaking such a formal analysis and utilizing data from prior crises, it is likely that considerable insight can be provided to the defense community on the manner in which crisis situations impact on the perceptions of key decision-makers, in terms of the options and alternatives they consider, the utilities which these decision-makers place over the set of available options, and the actions ultimately taken by these decision-makers.

Given such an insight, United States defense planners and decision-makers alike will be in a better position to act in two major areas:

o first, in the acquisition of system and implementation of policies which enhance capabilities to deal with crises in short time frames; and

o second, to structure the options it presents in a crisis to potential adversaries based on a sophisticated understanding of the manner in which decisions and actions are likely to be taken by such an adversary.

Looking at the range of crisis decisions which could confront United States decision-makers in the European theatre, the Middle East, and other regions, it is possible to envision a number of situations where the nature of the decision-process and the dynamic element would be critical to the formulation and presentation of United States options. In the opening phase of a NATO/Warsaw Pact conflict, for example, the timing of Soviet actions and United States/NATO responses could well serve to influence Soviet selection between moderation, Soviet selective nuclear targeting, theatre-level nuclear war, or implementation of a Soviet SIOP.[3]

Regardless of the region involved, it is clear that international crises share in common specific elements and problems for analysis. Thus, the problem becomes a general one of specifying specific crisis elements and developing a dynamic model for the analysis of crises which includes the question of timing as a fundamental variable, rather than as a characteristic of the crisis situation.

Based on the work of such theorists as Koopmans, Debreu and others, as well as previous work done by the investigators, the dynamic model will provide a formalized analysis of crisis choice situations within the context of the strategic situation perceived by the decision-makers, in terms of the salient variables.[4] By structuring the model in this fashion, it is then possible to consider:

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[3]An analysis of this problem is presented in a highly literate fashion in Benjamin S. Lambeth, *United States and Soviet Doctrinal Views on Selective Nuclear Targeting* (WN-9217-DDRE). Santa Monica, CA: The RAND Corporation, July 1976. See also, Lynn Etheridge Davis, "Limited Nuclear Options: Deterrence and the New American Doctrine," *Adelphi Papers No. 121*. London: International Institute for Strategic Studies, 1976.

[4]T. C. Koopmans, "Utility and Impatience," *Econometrica*, 28,2 1960, G. Debreu, "Representation of a Preference Ordering by a Numerical Function," in R. M. Thrall, C. H. Coombs and R. L. Davis (eds.) *Decision Processes*. New York: John Wiley & Sons, 1957. See also, G. Debreu, "Stochastic Choice and Cardinal Utility," *Econometrica*, 26, 3, 1958, and Karl H. Borch, *The Economics of Uncertainty*. Princeton, N.J.: Princeton University Press, 1968.

- changes in the critical crisis variables over the time frame involved;
- perceptions of such changes held over time by the key decision-makers;
- the discounted, normalized utilities of alternative options and strategies over time in the crisis.

Utilizing such a Bayesian approach, the dynamic analysis of the crisis decision process may be accomplished on a formal level and programmed for subsequent computer simulation and analysis of specific empirical situations.[5]

While previous efforts in the area of crisis management and the analysis of strategic decision-making have yielded both insight and data in many of the critical problem areas, it remains for the dynamic element to be added to the Bayesian deterrence calculus at a formal level and applied to existing data on specific international events and crisis.

Deterrence theory, as applied in a large number of studies, has sought to provide a logical framework for the analysis of statistical situations at specific points in time.[6] Through the use of an economic calculus, this theory has sought to determine the value, or utility of the possible options in a given situation for each of the parties involved. Operating on an assumption of rational choice for the decision-maker, the theory postulates that the alternative will be chosen which has the highest utility as perceived by the decision-makers. Although static in nature, this calculus provides the basic framework for a dynamic model.

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[5]See Central Intelligence Agency, *Handbook of Bayesian Analysis for Intelligence*. OPR-506 (June 1975). A number of related studies have been undertaken by CIA/OPR utilizing this approach, but cannot be cited on an unclassified basis.

[6]See, for example, Daniel Ellsberg, "The Crude Analysis of Strategic Choices," *RAND Monograph P-2183*. Santa Monica, CA: The RAND Corporation, 1960 ; C. E. Fink, "More Calculations About Deterrence," *Journal of Conflict Resolution*, 9,1, March 1965; J. E. Mueller, *Deterrence, Numbers and History*. University of California at Los Angeles, Security Studies Project, 1968; Bruce M. Russett, "The Calculus of Deterrence," *Journal of Conflict Resolution*, 7, 2, June 1963;and Glenn H. Snyder, *Deterrence and Defense: Toward a Theory of National Security*. Princeton, NJ: Princeton University Press, 1961.

1.2 Deterrence Theory and Crisis Analysis

The use of concepts from mathematics and utility theory has a long and variegated history in the literature on inter-nation deterrence.[7] In general, this application has been beneficial for two interrelated reasons. First, the use of mathematics necessitates rigorous thinking about variables and their inter-relationships. Second, the use of utility theory introduces a greater degree of precision into this rigorous thinking, a precision which lends additional clarity to discussion and leads to uncovering non-obvious relationships between related variables. Nevertheless, in applications to deterrence, five problem areas have arisen which diminish the value of results obtained thus far.

First, the concepts of utility theory that have been applied could have been applied with greater conformity to other applications, e.g., economics, voting,and electoral competition. For example, in these applications, it is assumed that a decision-maker first ranks the logically possible outcomes of a social situation from most to least preferred, and then constructs his utility function for outcomes.[8]

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[7] See, for example, C. F. Doran, "A Theory of Bounded Deterrence," *Journal of Conflict Resolution,* 17, 2:243-269 (1973); D. Ellsberg, "The Crude Analysis of Strategic Choices," in J. E. Mueller (ed.) *Approaches to Measurement in International Relations* , pp. 288-294. New York: Appleton-Century-Crofts (1960); C. F. Fink "More Calculations About Deterrence," *Journal of Conflict Resolution*, 9:54-65 (1965); D. E. Hunter, *Aspects of Mathematical Deterrence Theory,* Los Angeles: University of California, Security Studies Project (1971); J. E. Mueller (ed.) *Approaches to Measurement in International Relations*, New York: Appleton-Century-Crofts (1969); J. E. Mueller, *Deterrence, Numbers and History*. Los Angeles, University of California, Security Studies Project (1968); B. M. Russett, "The Calculus of Deterrence," *Journal of Conflict Resolution*, 7, 2:97-109 (1963); and J. D. Singer, *Deterrence, Arms Control and Disarmament: Toward a Synthesis in National Security Policy*. Columbus: Ohio State University Press (1962).

[8] A utility function is an assignment of real numbers to outcomes so as to reflect a decision-maker's preferences and his degree of preference for one outcome over another. See, J. S. Chipman "The Foundations of Utility," *Econometrica,* 28:193-224; R. D. Luce and H. Raiffa, *Games and Decisions,* New York: John Wiley and Sons (1957); W. H. Riker and P. C. Ordeshook, *An Introduction to Positive Political Theory,* Englewood Cliffs, NJ: Prentice Hall (1973) R.M. Thrall, C.H. Coombs, R.L. Davis (eds.), *Decision Processes*, New York: John Wiley and Sons (1954); and J. Von Neumann and O. Morgenstern *The Theory of Games and Economic Behavior*, Princeton: Princeton University Press (1944).

Deterrence theorists have usually jumped straight from the set of outcomes to a decision-maker's utility for outcomes. As a result of skipping the intermediate step (constructing preference orderings for outcomes), some theorists have generated unnecessary and confusing conceptual problems.[9]

Second, some concepts of utility theory, especially in a field called statistical decision theory, that have considerable relevance to mathematical deterrence theory have not been explicitly applied. These concepts relate to the problem of decision making under conditions of risk and uncertainty. While it is true that deterrence theorists have paid considerable attention to the problems engendered by risk and uncertainty, their discussions have been largely verbal and only indirectly related to the use of these concepts by statistical decision theorists. For example, deterrence theorists often talk of risk and uncertainty in terms of the possible responses of a decision-maker's opponent. The decision-maker assesses the probable responses of an opponent by estimating specific probability numbers for his possible courses of action.[10] On the other hand, in statistical decision theory, it is not only assumed that decision-makers estimate specific probability numbers for actions or outcomes, but also that decision-makers sometimes, perhaps due to a large measure of uncertainty, estimate a range of probability of numbers in which they

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[9]See, for example, C. F. Fink "More Calculations About Deterrence," who corrects, clarifies and refines B. M. Russett, "The Calculus of Deterrence."

[10]See D. Ellsberg "The Crude Analysis of Strategic Choices;" C. F. Fink, "More Calculations about Deterrence;" D. E. Hunter, *Aspects of Mathematical Deterrence Theory*; J. E. Mueller, *Approaches to Measurement in International Relations* and *Deterrence, Numbers and History*; B. M. Russett "The Calculus of Deterrence;" T. C. Schelling, *Arms and Influence*. New Haven: Yale University Press (1966); J. D. Singer, *Deterrence, Arms Control and Disarmament*; and G. H. Snyder *Deterrence and Defense: Toward a Theory of National Security*, Princeton: Princeton University Press (1961).

believe the true probability falls.[11] Thus, this approach is more general than that used by deterrence theorists up to now. Its relevance to mathematical deterrence theory is discussed subsequently.

Third, George and Smoke point out that a major problem in deterrence theory, especially in developing concrete, applicable policies, is a failure to clearly distinguish between the "levels" of deterrence.[12] George and Smoke define and discuss three levels of deterrence: deterrence of strategic war, e.g. between the United States and the Soviet Union; deterrence of limited wars, e.g. Korea and Vietnam; and deterrence of potential crises, e.g. Angola, Southern Africa. They argue that the major problem in development of efficatious deterrent policies is determining the appropriateness of a given policy for a given level. While these distinctions have been recognized by other deterrence theorists, we mention George and Smoke's discussion because we employ their levels of deterrence here. Aside from the possibility of escalation from one level to another, e.g. limited to strategic war, we note that, after including the concepts of risk and uncertainty from statistical decision theory, other important relationships, which have been only implicitly recognized by deterrence theorists between the levels emerge.

Fourth, there has been neglect of the dynamics of the deterrence situation, that is, variations over time in decision-makers' evaluations of outcomes and the factors relevant to such evaluations and estimates and how these variations and factors affect the deterrence situation.

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[11] See A. Rabushka and K. A. Shepsle, *Politics in Plural Societies, A Theory of Democratic Instability*. Columbus: Charles E. Merrill (1972); K. A. Shepsle "The Strategy of Ambiguity: Uncertainty and Electoral Competition," *American Political Science Review,* 66, 2:555-568 (1972); "Parties, Voters and the Risk Environment: A Mathematical Treatment of Electoral Competition Under Uncertainty," pp. 273-297 in R. G. Niemi and H. F. Weisberg (eds) *Probability Models of Collective Decision Making*. Columbus: Charles E. Merrill (1972); and Kenneth Shepsle, "Essays on Risky Choice in Electoral Competition," Ph.D. dissertation, Rochester: University of Rochester (1970).

[12] See A. L. George and R. Smoke, *Deterrence in American Foreign Policy: Theory and Practice*. New York: Columbia University Press (1974) pp. 38-57.

This neglect is, of course, not entirely due to lack of effort by deterrence theorists, though perhaps they can be faulted for lack of logical completeness in formulation of the statics of deterrence, but to the complexity of the abstract logic involved, a complexity suggested in our concluding section.

Last, there is the problem of the empirical applicability of mathematical deterrence theory. Again, deterrence theorists cannot be faulted for lack of effort, and the difficulties of application are not surprising, given the relative complexity and abstractness of the mathematics involved. We do not propose to deal with this or the preceding problem. However, a few hueristic examples are offered to illustrate key points in our analysis, and we very briefly discuss the dynamics of deterrence and empirical application in closing.

Adopting George and Smoke's three levels of deterrence as a general framework for analysis, the paper focuses on the first two problem areas - namely, it presents a more direct application of utility theory to deterrence and introduces some additional concepts from *statistical decision* theory to more clearly delineate the statics of the deterrence situation.

II. THE CALCULUS OF DETERRENCE REVISITED

2.1 Some Basic Assumptions

Following the lead of earlier theorists, we first outline some fundamental assumptions used in subsequent discussion. We recognize that these assumptions are an oversimplification of reality, but they are useful starting points for analysis. The assumptions we employ to relate to the actors are involved in the deterrence situation itself.

First, we assume that there are only two actors (but we do allow "sub-actors", e.g., client states) who are called either deterrer and deterree or defender and aggressor. In the case of nation states, this assumption does entail an element of false personification, especially, as noted by George and Smoke, where they analyze the actual development of deterrent policies.[13] However, at this stage we choose to postpone the consideration of such problems.

Second, we consider the deterrence situation only from the potential aggressor's point of view. For the moment we postpone consideration of possible defender responses to an aggressor's actions and the aggressor's explicit estimates of possible defender responses.[14] However, we do discuss the factors relevant to the development of deterrent policies by the defender. Thus, the decision problem is simplified by focusing on the process through which a potential aggressor decides to adopt (e.g. start a war) or not adopt (e.g. not start a war) some particular course of action.

Third, we assume that the actors are rational decision makers. As mentioned, the defender's decision calculus is not dealt with, but we do assume that he is interested in developing policies that deter the potential aggressor from adopting some course of action, e.g. starting a war, that the defender views as undesirable. Regarding the potential

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[13] George and Smoke, *Deterrence in American Foreign Policy*, pp. 281-288.

[14] For one discussion of this situation, see Mueller, *Deterrence, Numbers and History*, pp. 15-16.

aggressor, our assumption of rationality is more complicated. First, we assume that he can rank the possible outcomes of the deterrence situation from most to least preferred and that a utility function can be constructed on his preference ordering. Second, we assume that he is sufficiently intelligent to estimate the probabilities of various outcomes, contingent upon each possible course of action, and that his probability estimates may be either specific real numbers, or some range of numbers between 0 and 1 inclusive. Third, we assume that he can relate (through his probability estimates) the possible courses of action open to him to the possible outcomes in such a way as to establish his preference ordering for actions. Last, in choosing a course of action, we assume that he chooses that action which, *ceteris paribus*, yields him the highest expected utility. We now explore the specifics of these assumptions in each of our three levels of deterrence.

2.2 Deterrence of Strategic War

While recognizing that modern strategic war would likely be fought with nuclear weapons alone, we nevertheless define strategic war as an attack with conventional forces or nuclear weapons or some combination of the two on one actor's homeland by the other, and such an attack may or may not entail significant retaliation on the attacker, depending upon the degree of success of the attacker's first strike.[15]

In the simplest situation, there are three possible outcomes of a strategic war:

(1) Victory, V, defined as a completely successful first strike such that the victim cannot retaliate at all on the attacker;

(2) Defeat, D, defined as an unsuccessful first strike such that the victim is able to retaliate on the attacker, and damage may range from minimal or acceptable (perhaps in the mind of the attacker?) to complete destruction;

(3) Status Quo, SQ, which is the outcome that obtains if

[15] The discussion of this section draws largely on those of D. Ellsberg, "The Crude Analysis of Strategic Choice;" D. E. Hunter, *Aspects of Mathematical Deterrence Theory*, J. E. Mueller, *Approaches to Measurement in International Relations*, and *Deterrence, Numbers and History*.

the potential agressor chooses not to initiate strategic war.

In order to estimate the likelihood that a potential aggressor is deterred from initiating a strategic war, we must first discover his preference ordering for outcomes. In general, the construction of his preference ordering requires the potential aggressor to weigh the benefits versus the costs of each outcome. For example, V might give him world hegemony. However, costs would be incurred in his expenditures of scarce resources required to build sufficient first strike forces or possibly retaliatory damage to sub-actors (satellite countries). However, we assume that the potential aggressor, in his own mind, can weigh these benefits and costs and arrive at a preference ordering for the outcomes. At this point, we avoid specification of factors entering into his benefit-cost calculations because their complexity places them far beyond the scope of this paper and they are more appropriately dealt with in discussion of empirical applications. Thus, as displayed in Table 1 and ignoring indifference, there are six possible preference orderings of the three outcomes, and we assume that, after completing his benefit-cost calculations, the potential aggressor can select one of these orderings as his own.

Next, we need to construct the potential aggressor's utility function on the set of outcomes. Let us suppose that he has determined his preference ordering to be I. Following applications in other fields, we assign the largest number to his most preferred outcome, the smallest to his least preferred, and we use the standard "0, 1-normalization", which means that the largest number used is 1, the smallest 0, and the number assigned to the middle-ranked outcome falls in between.[16] The result of such an

[16] See J. S. Chipman, "The Foundations of Utility," *Econometrica* (1960) 28:193-224; R. D. Luce and H. Raiffa, *Games and Decisions*; A. Rabushka and K. A. Shepsle, *Politics in Plural Societies*; W. H. Riker and P. C. Ordeshook, *An Introduction to Positive Political Theory*; and J. Von Neumann and O. Morgenstern, *The Theory of Games and Economic Behavior*.

TABLE 1

Order of Preference	I	II	III	IV	V	VI
1	V	V	D	D	SQ	SQ
2	SQ	D	V	SQ	D	V
3	D	SQ	SQ	V	V	D

TABLE 2

Order of Preference	Outcome	Utility Function (U)
1	V	$U(V)=1$
2	SQ	$U(SQ)=k_1,\ 0<k_1<1$
3	D	$U(D)=0$

TABLE 3

Preference Ordering	U(V)	U(SQ)	U(D)	U(SQ)>Pr(V)?
I	1	k_1	0	depends
II	1	0	k_2	never
III	k_3	0	1	never
IV	0	k_1	1	depends
V	0	1	k_2	always
VI	k_3	1	0	always

assignment is displayed in Table 2.

There are two possible courses of action open to the potential agressor: (1) not initiate a strategic war (A_1), or (2) initiate a strategic was (A_2). If he selects A_1, he does not begin a strategic war and his "payoff" is U(SQ), which includes his estimate of the probability of an attack by the defender. If he selects A_2, the situation is more problematic. Recalling our definition of V, we see that an element of uncertainty is present as the potential agressor may not, prior to selection of A_2, know what the probability of obtaining V, Pr(V), actually is. We can reasonably suppose that it is less than unity, though it may be quite large. Thus, a choice of A_2 entails a lottery in which the potential agressor has some probability which, for the moment, we assume he can estimate as a specific real number between 0 and 1, of obtaining his most preferred outcome V, but also some probability (in this case, 1-Or(v)) of realizing his least preferred outcome. The dilemma can be resolved by assuming that he chooses that action which maximizes his expected utility (EU), Here,

$$EU(A_1) = U(SQ)$$

and

$$EU(A_2) = Pr(v) \times U(V) + (1-Pr(V)) \times U(D).$$

The potential agressor than is deterred, that is, selects A_1, if and only if

$$EU(A_1) > EU(A_2).$$

Deterrence fails if and only if

$$EU(A_1) < EU(A_2).$$

If the two expected utilities are equal, deterrence may succeed or fail depending upon factors other than those in the immediate calculation, e.g., manipulation by the defender of Pr(V) and/or the benefit-cost calculations of the potential aggressor.

Examining the condition for successful deterrence and substituting from Table 2, we have

$$\begin{aligned} EU(A_1) &> EU(A_2), \\ U(SQ) &> Pr(V) \times U(V) + (1-Pr(V) \times U(D), \\ k_1 &> Pr(V) \times 1 + (1-Pr(V)) \times 0, \\ k_1 &> Pr(V). \end{aligned}$$

Thus, deterrence succeeds if and only if U(SQ)> Pr(V). At this point, an age old problem regarding deterrence, namely, we usually known when deterrence fails, but we don't always know when (or why) it succeeds, becomes relevant. Specifically, more than one preference ordering is consistent with U(SQ) >Pr(V).[17]

Table 3 displays the results of calculations for all preference orderings of Table 1 and their attendant utility functions. We note that in two cases (V and VI) deterrence always succeeds regardless of the potential agressor's estimate of Pr(V); in two cases (II and III) deterrence never succeeds; and in two cases (I and IV) deterrence may or may not succeed depending upon the potential agressor's utility for SQ and his estimate of Pr(V).[18]

Thus, successful deterrence of a potential agressor depends upon three fundamental factors: (1) his preference ordering for the possible outcomes of the deterrence situation; (2) if and only if SQ is his middle ranked outcome, the relative valuations of SQ compared to V and D, that is, assuming preference ordering I, the relative magnitudes of u(V) - U(SQ) and U(SQ) - U(D); (3) his estimate of Pr(V). Similarly, the defender, in his efforts to develop successful deterrent policies, must address four basic questions: What is the potential agressor's preference ordering for the possible outcomes of the deterrence situation and how can I influence that ordering? If I discover SQ to be his middle ranked outcome, how can I influence his evaluation of SQ versus V and D? How can I influence his estimate of Pr(v)? How will dealing with one question affect answers to the others, that is, how are the factors involved interrelated?

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[17]This point is made by C. F. Fink "More Calculations About Deterrence," and J. E. Mueller, *Approaches to Measurement in International Relations* and *Deterrence, Numbers and History*.

[18]Preference orderings III and IV are, empirically, a bit perverse. Writing in a similar vein, J. E. Mueller, *Approaches to Measurement*, pp. 284-285 notes: "Suppose...that the attacker is a masochist who prefers defeat to victory...Then to deter, one wishes to make the probability of victory as high as possible."

2.3 Deterrence of Limited War

Here, the abstract logic and calculations of the preceding section apply. The major difference concerns the definitions of limited war, V and D. The problem we encounter is one similar to that found in the definition of strategic war, but magnified; namely, the ambiguity of the term. At times it is easier to say what limited war is not rather than what it is. We would not call a war limited if one actor attacked the other's homeland, e.g. a Soviet attack on the United States. Examination of the goals an actor has at the outset of a limited war is not particularly helpful either, e.g. George and Smoke discuss at length how American objectives in Korea changed as the war progressed.[19]

Keeping these difficulties in mind, we define a limited war as one in which neither homeland is attacked with nuclear and/or conventional weapons, and the major *a priori* goal of the initiator of the war is to eliminate one or more of the other actor's client states. Thus, although it might quickly escalate into strategic war, a Soviet invasion of Western Europe would fall under this definition as well as relatively less serious aggressions, e.g. Korea V is defined as the achievement by the initiator of his *a priori* goal(s). D is defined as the failure by the initiator to achieve his *a priori* goal(s). Thus, paradoxical as it may seem, both the United States and the People's Republic of China "won" the Korean War. As George and Smoke note, the initial American goal was to preserve North Korea and, after considerable human and material losses on both sides, this was the outcome.

2.4 Deterrence of Potential Crises

Again, the abstract logic and calculations of the section on strategic war apply, but, as in the case of limited war, definitions of key terms differ and, unfortunately, make the situation even fuzzier.[20] We define a

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[19] See A. L. George and R. Smoke, *Deterrence in American Foreign Policy: Theory and Practice*. New York: Columbia University Press (1974) pp. 38-57.

[20] Our discussion here draws on those of C.F. Fink, "More Calculations About Deterrence;" and B. M. Russett, "The Calculus of Deterrence."

crisis as a manifest threat by one actor against the other and/or one or more of that actor's clients and the major, *a priori* goal(s) of a threatener is (are) to obtain from the other actor and/or his client(s) some tangible concession(s) or intangible payoff(s), in the threatener's mind, of an economic, political, and/or military nature. A threat may be either verbal or non-verbal and can also be multifaceted. For example, in the first Berlin crisis (1948-1949), it appears that a major, *a priori* goal of Soviet action in closing ground access routes to the city was to force the Western Allies to abandon their position there.[18] By our definition, this Soviet action constituted a threat to a United States client.

Regarding the other key terms, we define A_1 as not issuing a threat. A_2 is defined as issuing a threat. V is defined as achievement by a threatener of his major, *a priori* goals(s). D is defined as a threatener's failure to achieve his major, *a priori* goal(s). SQ is defined as the outcome that obtains if a potential threatener does not issue a threat. Pr(v) is defined as the potential threatener's estimate that his threat will successfully achieve his major, *a priori* goals.

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[21] See A. L. George and R. Smoke, *Deterrence in American Foreign Policy: Theory and Practice*. New York: Columbia University Press (1974) pp. 107-139.

III. RISK AND UNCERTAINTY

3.1 Risk and Uncertainty in Deterrence Theory

Deterrence theorists have devoted considerable, but largely verbal, attention to the effects of "risk" and "uncertainty" on the calculations outlined in the previous section.[22] In general, the terms have applied to decision makers' benefit-cost calculations, probability estimates and, given selection of a specific course of action, evaluation of the possible response of an opponent and probability estimates of the likelihood of these various possible responses. For example, assuming preference ordering I at the level of strategic war, if a potential aggressor selects A_1, which gives him U(SQ), one "risk" he incurs is the chance that the defender will then select A_2. If a potential aggressor contemplates selection of A_2, which gives him $EU(A_2) = Pr(V) \times U(V) + (1-Pr(V)) \times U(D)$, then the "uncertainty" attendant upon such a decision is the discrepancy between Pr(V), a subjective estimate based at least in part on the information, which may or may not be complete or correct, available to him, and the "true" value of Pr(v) if he actually selects A_2.

Thus, although they are not used in a mutually exclusive manner, risk usually refers to a potential aggressor's assessment of the defender's possible responses and their probabilities, given his own choice of an action. Uncertainty usually refers to the magnitude of the deviation from the "true" values of a potential aggressor's probability estimates of various outcomes resulting from an action he himself may adopt. In this context, the manipulation of risk and uncertainty by a defender, presumably resulting in the potential aggressor's selection of A_1, usually refers to influence on the potential aggressor's benefit-cost calculations and the specific probability estimates he makes.

[22] The discussion here is based mainly on those of B. Brodie, *Strategy in the Missile Age*, Princeton: Princeton University Press (1959); H. Kahn, *On Escalation: Metaphors and Scenarios*. New York: Praeger Publishers (1965); T. W. Milburn, "What Constitutes Effective Deterrence," *Journal of Conflict Resolution*, 3, 2:138-145; T. C. Schelling, *Arms and Influence*, and *The Strategy of Conflict*; J. D. Singer, *Deterrence, Arms Control and Disarmament*, and "Inter-Nation Influence: A Formal Model;" G. H. Snyder "Deterrence and Power," and *Deterrence and Defense: Toward a Theory of National Security*.

3.2 Risk and Uncertainty in Statistical Decision Theory

In statistical decision theory, the ideas of risk and uncertainty are used in a manner similar to deterrence theorists.[23] While deterrence theorists are largely verbal in their discussion, however, statistical decision theorists translate such discussion into explicit mathematical constructs and introduce additional relevant concepts.

Two basic questions addressed in statistical decision theory are relevant to deterrence theory: Is the decision maker, by nature, before he is placed in a specific decision situation, a gambler or not - that is, is he disposed to accept risks (risk acceptant), avoid them (risk averse) or equally disposed (risk neutral) toward the risks that may inhere in various decision situations? Secondly, does the decision maker, regardless of whether or not he is risk averse, neutral or acceptant, make specific probability estimates of the possible outcomes of his actions, or does he estimate some range of numbers between 0 and 1 in which he believes the "true" probability to lie?

The significance of the first question, which, as is seen subsequently, implies a somewhat different interpretation of risk than current deterrence theory, lies in the fact that different decision makers, faced with the identical decision situation (e.g. prefernce ordering and probability estimates), will make different choices depending upon whether or not each is risk averse, neutral or acceptant. The significance of the second question, which implies a somewhat different interpretation of uncertainty than current deterrence theory, lies in the fact that two decision makers, both of whom are risk averse, neutral, or acceptant, and faced with the identical decision situation will make different choices depending upon whether or not their probability estimates are specific numbers or some range of values.

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[23] The discussion of this and the next section is based mainly on those of A. Rabushka and K. A. Shepsle, *Politics in Plural Societies;* and K. A. Shepsle, "The Strategy of Ambiguity," "Parties, Voters and the Risk Environment," and "Essays on Risky Choice and Disarmament."

3.3 Statistical Decision Theory and Deterrence Theory

The general comments of the preceding section can be made more explicit by applying them to the earlier calculations on the deterrence of strategic war.[24] Let us suppose that a potential aggressor has selected preference ordering I and is contemplating selection of A_2, which means he must evaluate a lottery beween V and D. The key elements in this evaluation are his estimate of Pr(V) and the (expected) utility that this estimate generates. In short, he must evaluate the lottery (Pr(V), V; (1-Pr(V), D). We note that, as Pr(V) varies from 0 to 1, a series of lotteries is generated which vary from (0, V; 1, D) to (1, V; 0, D) and, by the assumption of preference ordering I, the latter is his most preferred lottery and the former his least preferred. Here, it is possible to construct his utility function for all lotteries involving V and D, and, by the assumption of preference ordering I, U(0, V; 1, D) = 0 and U(1, V; 0, D) = 1. Further, we know that, since SQ is his middle ranked outcome, U(SQ) must be between 0 and 1 and equivalent to the utility of some lottery between (0, V; 1, D) and (1, V; 0, D), that is, for some Pr(V), say Pr(V)*, 0 < Pr(V)* < 1, U(SQ)-U(PR(V)*, V; (1-Pr(V)*), D).

As mentioned, the first key question is: Is the decision maker a gambler or not? The terms used are risk averse for the non-gambler, risk neutral for the indifferent (as explained below) decision maker and risk acceptant for the gambler. Each of these personality types is distinguished by the shape of his utility function on lotteries involving the basic outcomes. In this context, there are three general shapes. A risk averse (RAV) decision maker have a concave utility function defined on lotteries ranging from (0,V; 1, D) to (1, V; 0, D); a risk neutral

[24] Our discussion here has much in common with C. F. Doran, "A Theory of Bounded Deterrence." His analysis concerns one, very specific set of circumstances; ours is more general and thus further removed from the real world. His use of the terms risk and uncertainty is remarkably similar to our own, although he nowhere cites Rabushka and Shepsle, *Politics in Plural Societies;* Shepsle, "The Strategy of Ambiguity," "Parties, Voters and the Risk Environment," "Essays on Risky Choice in Electoral Competition," or relevant literature in statistical decision theory.

FIGURE 1

ALTERNATIVE RISK FUNCTIONS

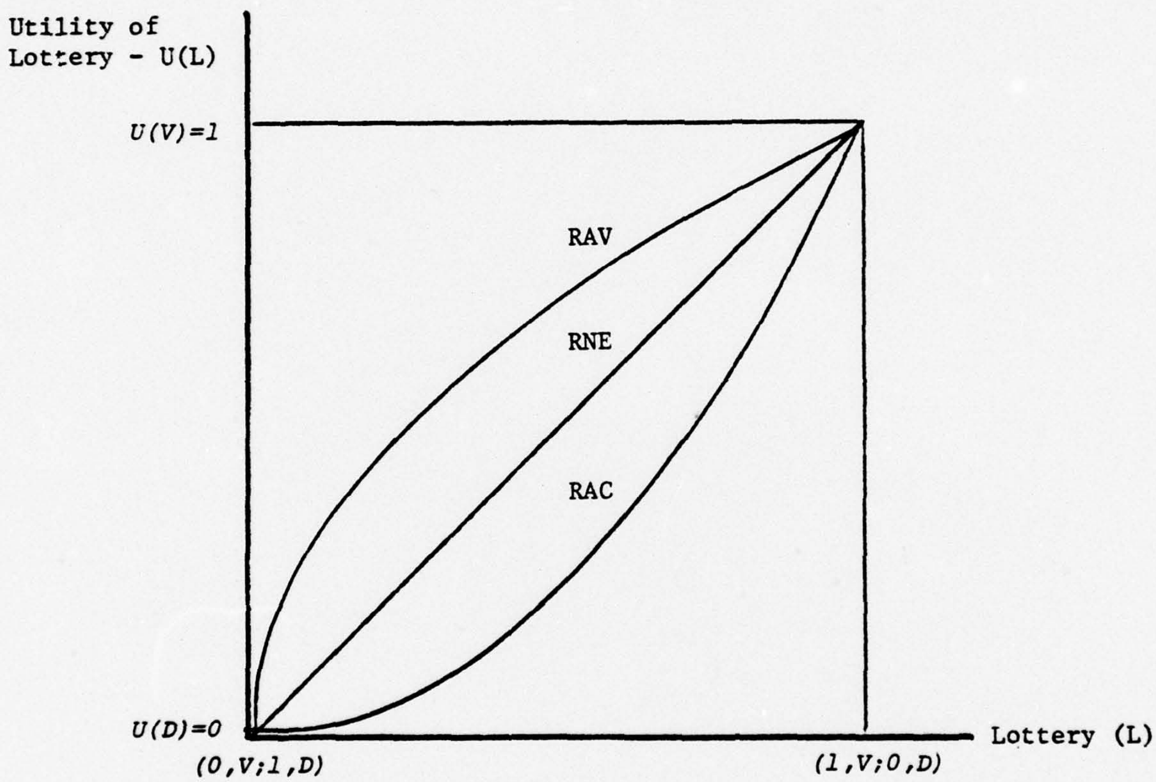
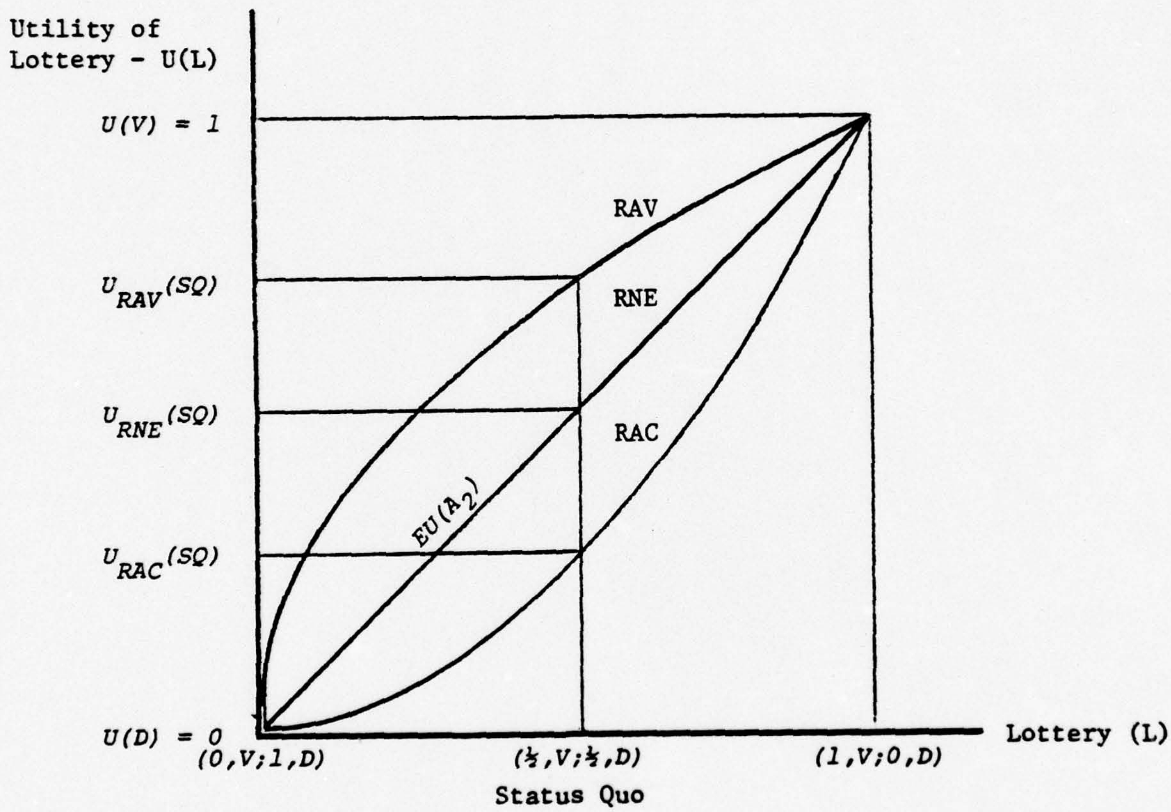


FIGURE 2

STATUS QUO UTILITIES



(RNE) decision maker a linear utility function; and a risk acceptant (RAC) decision maker a convex utility function. These three types, with appropriate labels, are displayed in Figure 1.

The significance of the distinction between RAV, RNE, and RAC decision makers resides in the fact that, faced with the identical decision situation, each behaves in a different manner. Suppose, for the moment, that we have three potential aggressors, all with preference ordering I, and each makes the same estimate of Pr(V)-1/2, but one is RAV, another RNE and the third RAC. Further, suppose that, for all three, U(SQ)=U(1/2. V; 1/2, D). Recall our condition for successful deterrence and the decision rule for selection of an action -namely, a potential agressor is deterred if and only if $EU(A1) > EU(A_2)$ and he always chooses that action which, *ceteris paribus,* gives him the greatest expected utility. In Figure 2 we see that the expected utility of each lottery resulting from selection of A_2 with varying estimates of Pr(V) is the chord connecting the points [(0, V; 1, D), 0] and [(1, V; 0, D), 1], or equivalent to the utility function for an RNE decision maker, and we have, for the RAV decision maker,

$$EU(A_1) = U_{RAV}(SQ) > EU(A_2) = EU(1/2, V; 1/2, D) = 1/2 \times U(V) + 1/2 \times U(D);$$

for the RNE decision maker,

$$U_{RNE}(SQ) = EU(1/2, V; 1/2, D);$$

and, for the RAC decision maker,

$$U_{RAC}(SQ) < EU(1/2, V; 1/2, D).$$

Thus, the RAV decision maker prefers A_1 to A_2; the RNE decision maker is indifferent between A_1 and A_2; and the RAC decision maker prefers A_2 to A_1. Another way of stating this result is that the RAV decision maker prefers the "certain" outcome SQ to the more "risky" lottery option (1/2, V; 1/2, D). that is, he is "averse" to taking the risks inherent in lotteries. The RNE decision maker is indifferent between the certain outcome and the lottery option -that is, he is "neutral" when required to select between a certain outcome and a lottery option. The RAC decision maker prefers the lottery option to the certain outcome -that is, he is "acceptant" of the risks inherent in lotteries.

The second key question is: Does the decision maker make specific probability estimates or does he estimate some range of numbers between 0 and 1 in which he believes the "true" probability to lie? In extant deterrence theory as well as our discussion up to now, it is implicitly assumed that all decision makers, regardless of the process used in making the calculation, estimate probabilities, but most especially Pr(V), as specific real numbers, e.g. Pr(V) = 1/2. On the other hand, statistical decision theory as developed by Shepsle deals explicitly with situations in which decision makers may estimate some range of probability numbers in which they believe the "true" value to lie.

For example, suppose during his decision process a potential aggressor says to himself: "Based on the information I have about my capabilities vis-a-vis the capabilities and intentions of the defender, I don't think I can come up with a specific estimate of the probability of victory if I launch a first strike." In other words, he can not decide (he is uncertain) which specific lottery will result from selection of A_2. "However, it seems to me that the probability may fall between two particular numbers." In other words, he believes that the "true" value of Pr(V) and the resulting "true" lottery fall in some range, e.g. the interval [a; b], where a and b are specific values of Pr(V), which generate specific lotteries, and $a < b$. "For lack of any better information, suppose every lottery is equally likely to occur." In other words, he believes that, if he selects A_2, all lotteries in [a; b] are equally likely to obtain. This estimate is called a "risk function," R(L), and is a probability distribution over lotteries. Hence, R(L) conforms to the mathematical condition required of all such distributions, namely,

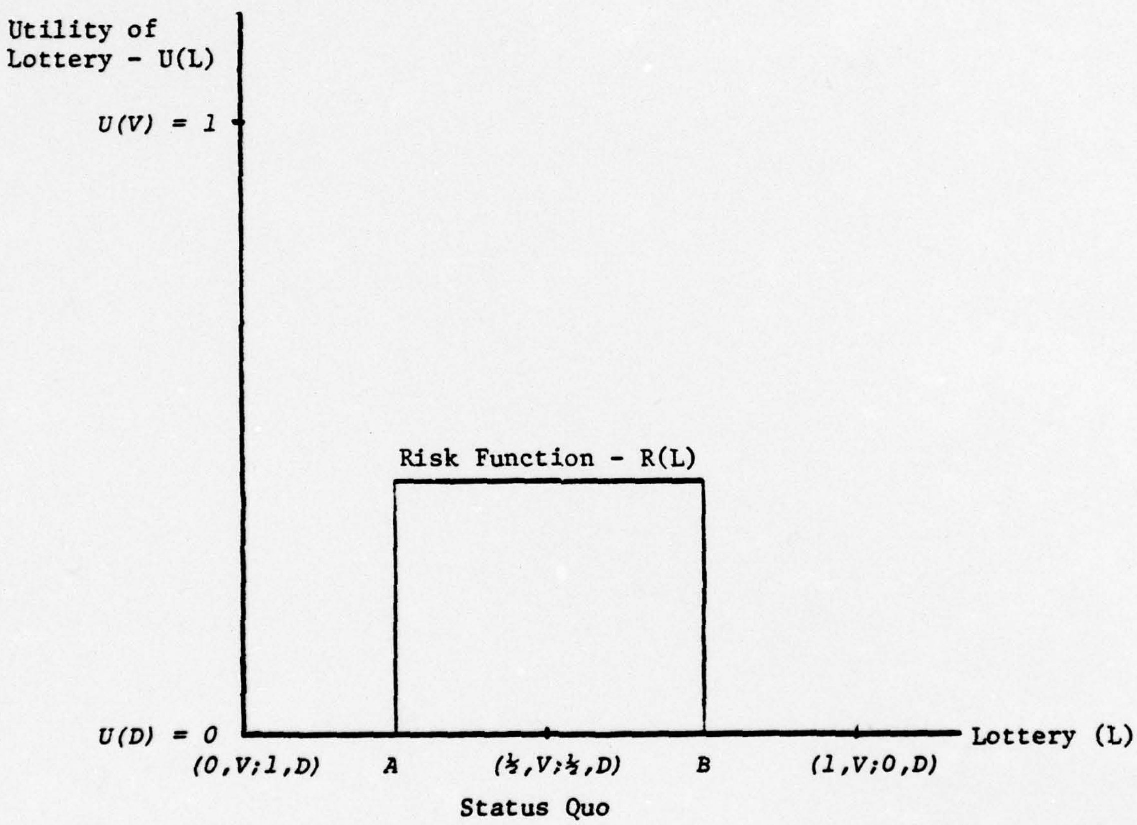
$$\int_a^b R(L)dL=1.$$

These concepts are displayed graphically in Figure 3.

"Having estimated the probability of victory this way, how can I decide between striking and waiting?" We note that this decision situation is rather different than earlier. A_1 remains the decision not to initiate strategic war,

FIGURE 3

RISK FUNCTION FOR THE STATUS QUO



which, since $EU(A_1) = U(SQ)$, still leads to the payoff $U(SQ)$. On the other hand, selection of A_2, which still means initiating strategic war, leads not to a specific lottery between V and D, but a set of lotteries each of which occurs between some estimated probability, as given by $R(L)$, and the expected utility calculation is not straightforward. However, although much more complex, the decision maker can still calculate $EU(A_2)$. Shepsle has formally proven that, in situations such as these, the expected utility of a course of action can be obtained by multiplying the decision maker's estimated risk function by his utility function and integrating the product over the range of the risk function. Thus,

$$EU(A_2) = \int_a^b R(L)U(L)dL.$$

Since it is constant over [a; b), $R(L)$ is equal to $1/(b-a)$, and the calculation becomes

$$EU(A_2)=1/(b-a) \int_a^b U(L)dL.$$

The significance of this approach to probability estimates lies in the fact that two decision makers, both of whom are risk averse, neutral or acceptant and faced with the same decision situation, make different choices depending on which type of probability estimate they make. However, let us return momentarily to the preceding analysis. Examining Figure 4, which is Figure 3 superimposed on Figure 2, we note that SQ, the certain outcome, is located at the midpoint of interval [a; b].[25] Adapting Shepsle's approach, we find it is still the case that, for the RAV decision maker,

$$EU(A_1)=U_{RAV}(SQ) > EU(A_2)=1/(b-a) \int_a^b U_{RAV}(L)dL;$$

[25] Unfortunately, for Shepsle's logic to apply, these restrictive conditions must be met, namely, $R(L)$ constant and SQ located at the midpoint of [a;b]. If these restrictions are dropped, it does not mean that the logic no longer applies, but that the analysis entails a degree of complexity beyond the scope of this paper.

FIGURE 4

RISK FUNCTION AND STATUS QUO UTILITIES

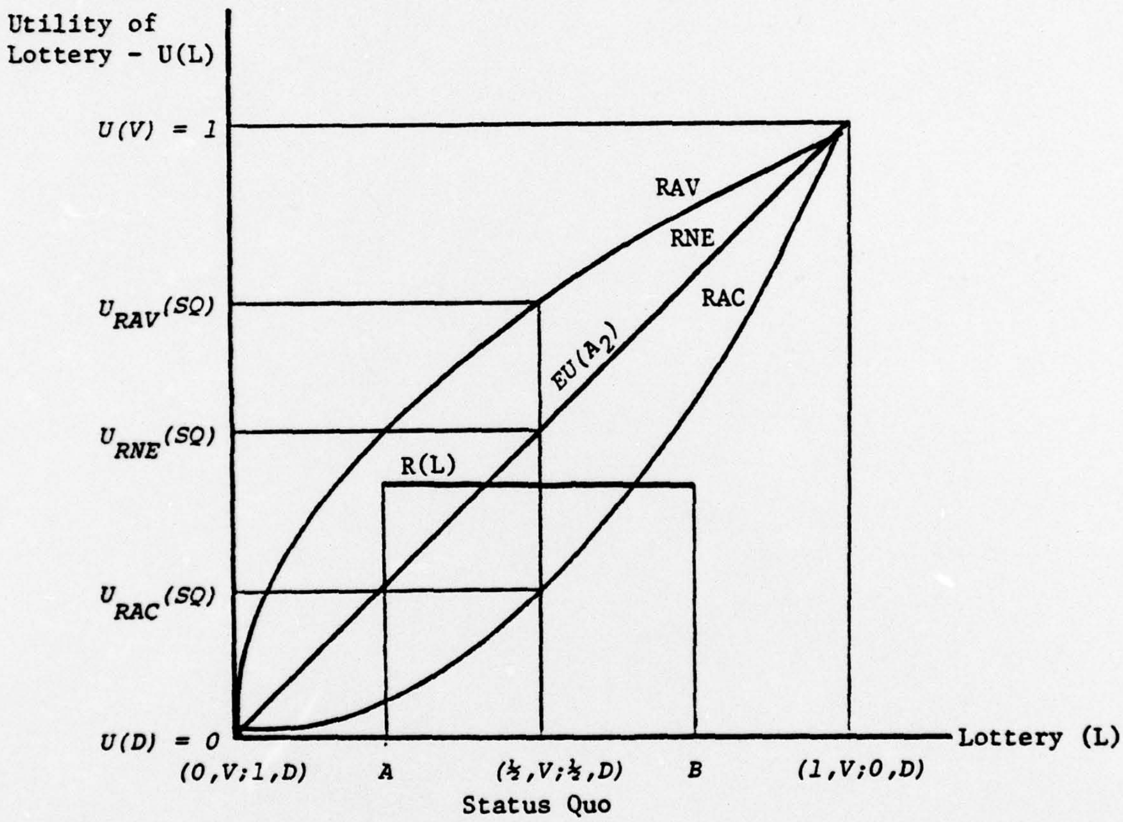
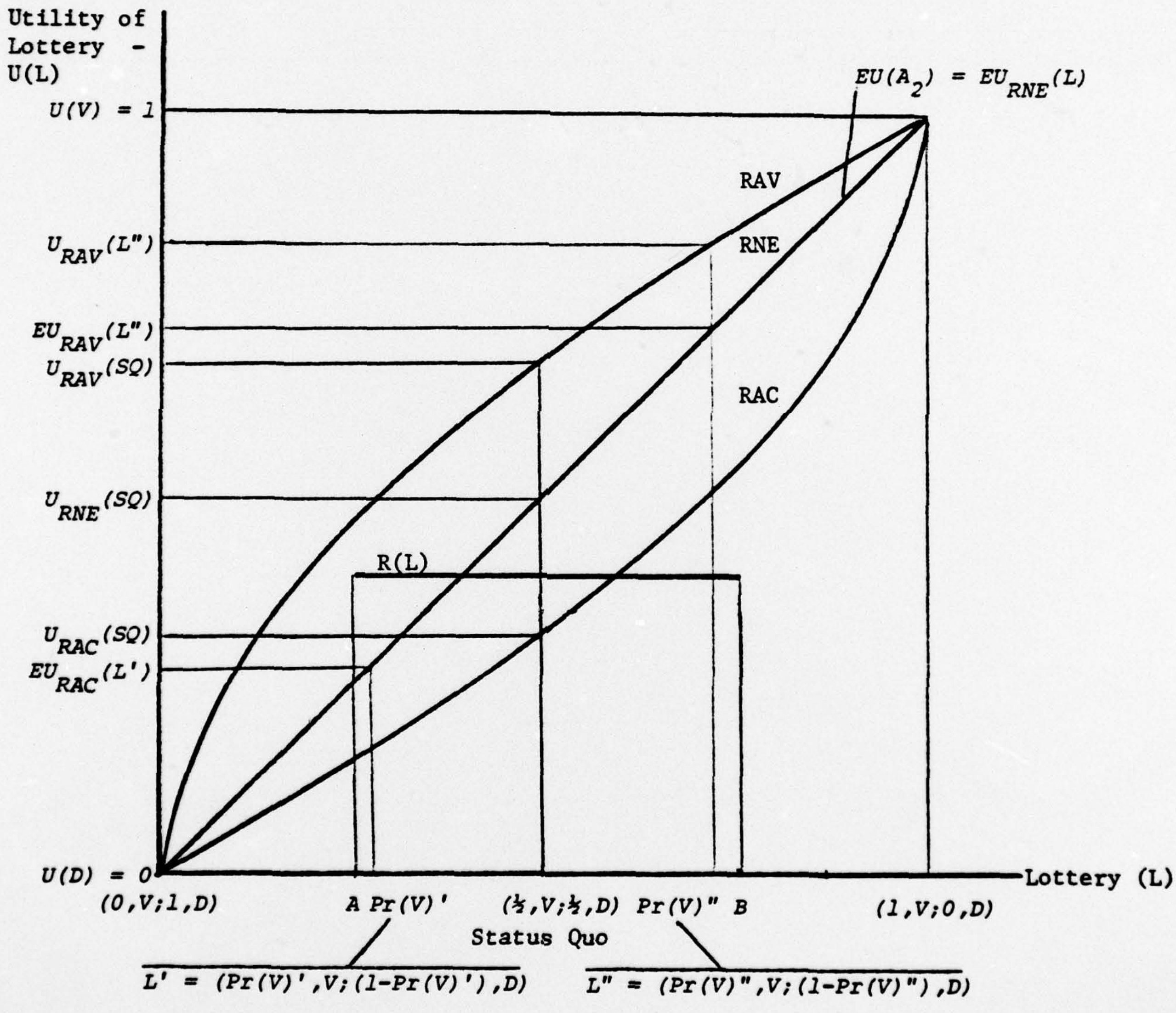


FIGURE 5

ALTERNATIVE PROBABILITY ESTIMATES FOR STATUS QUO



for the RNE decision maker,

$$U_{RNE}(SQ) = 1/(b-a) \int_a^b U_{RNE}(L)dL;$$

and, for the RAC decision maker,

$$U_{RAC}(SQ) < 1/(b-a) \int_a^b U_{RAC}(L)dL.$$

To illustrate the impact of the different types of probability estimates, consider Figure 5. First, let us suppose that some RAC decision maker estimates Pr(V) to be the specific number Pr(V)', which generates the lottery L'=(Pr(V)', V; (1-Pr(V)'), D). We see that $U_{RAC}(SQ) > EU_{RAC}(L')$ and so deterrence succeeds - the decision maker selects A_1. Suppose another RAC decision maker estimates PR(V) to be R(L), then

$$EU_{RAC}(A_2)=1/(b-a) \int_a^b U_{RAC}(L)dL > U_{RAC}(SQ)$$

and deterrence fails - the decision maker selects A_2.

Next, suppose some RAV decision maker estimates Pr(V) to be Pr(V)", which generates lottery L"=(Pr(V)", V; (1-Pr(V)"),D). Here, we see that $EU_{RAV}(L") > U_{RAV}(SQ)$ and deterrence fails. Suppose another RAV decision maker estimates Pr(V) to be R(L), then

$$EU_{RAV}(A_2)=1/(b-a) \int_a^b U_{RAV}(L)dL < U_{RAV}(SQ)$$

and deterrence succeeds.

Last, suppose some RNE decision maker estimates Pr(V) to be Pr(V)', then we see that $EU_{RNE}(L') < U_{RNE}(SQ)$ and deterrence succeeds. However, if he estimates Pr(V) to be Pr(V)", then $EU_{RNE}(L") > U_{RNE}(SQ)$ and deterrence fails. If some other RNE decision maker estimates Pr(V) to be R(L), then

$$EU_{RNE}(A_2)=1/(b-a)\int_a^b U_{RNE}(L)\,dL=U_{RAV}(SQ)$$

and deterrence may succeed or fail depending upon factors other than those involved in the immediate calculation.[26]

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[26] We note here that both Pr(V)' and Pr(V)" fall in the interval [a;b]. However, this does not necessarily have to be the case, but Pr(V)' must be either less than a or greater than but close to a, that is, it cannot be "too much" greater than a. The upper limit of Pr(V)' depends upon the RAC decision maker's estimate of the size of interval [a;b] and the degree of convexity of his utility function. Analogous reasoning holds for Pr(V)", its lower bound, b and the degree of concavity of the RAV decision maker's utility function.

IV. CONCLUSION

In this paper we have examined some theoretical difficulties present in extant mathematical deterrence theory. Adopting a distinction formulated by George and Smoke regarding the actual levels of deterrence, we first noted that concepts from utility theory already applied to the logic of deterrence could have been applied more directly and thus with greater clarity. We then proceeded to demonstrate the relevance of some additional concepts from utility and statistical decision theories and their impact on the calculations of rational decision makers. Aside from a few hueristic examples, we avoided consideration of the logic's empirical applicability. Specifically, we did not probe into the factors relevant to decision makers' benefit-cost calculations or probability estimates. In our opinion, such considerations are best left for future work which, as a result of our efforts here, we believe will rest on firmer theoretical foundations.

Aside from empirical considerations, some extensions of our formulations are immediately obvious. For example, if we allow decision makers to express indifference between the various basic outcomes, then, as shown in Table 4, the set of logically possible preference orderings expands from six to thirteen. Further, if we wish to evaluate the logical conditions relevant to the success or failure of deterrence in an international system with only two principal actors, then, initially, 91 different pairs of these preference orderings must be examined. Adding the other factors discussed - that is, are decision makers risk averse, neutral, or acceptant, what type of probability estimates do they make and what level of deterrence is relevant, we must examine, before elimination of redundant and trivial cases, 9828 logically possible deterrence situations.

In this context, the level of deterrence is an especially important variable as it is logically possible for the same decision maker to be risk averse at one level, risk neutral at another and risk acceptant at the third. For example, we can reasonably argue that both American and Soviet decision makers are risk averse at the level of strategic war. The fact that neither set has initiated such a war constitutes some, albeit ambiguous, evidence for this proposition. On the other hand, we can also

TABLE 4

Preference Ordering	U(V)	U(SQ)	U(D)	U(SQ)>Pr(V)?
I	1	k_1	0	depends
I'	1	0	0	never
I"	1	1	0	always
II	1	0	k_2	never
II'	1	0	1	never
III	k_3	0	1	never
III'	1	0	0	never
IV	0	k_1	1	depends
IV'	0	1	1	always
V	0	1	k_2	always
V'	0	1	0	always
VI	k_3	1	0	always
VII	1	1	1	depends

contend that both sets of decision makers could simultaneously be risk acceptant at the limited war and potential crisis levels. Spectacular failures of deterrence (.e.g. the outbreak of the Korean War) and the initiation of crises (e.g. deepening American involvement in Vietnam and Soviet actions in Cuba) by both sets provide some evidence for this statement. Nevertheless, the *a priori*, theoretical relevance of these distinctions remains and requires, along with our other formulations, further theoretical and empirical investigation. This paper is best regarded as another step in the direction of theory building.

APPENDIX

A. Strategy of Application

At this early stage of the project, specific procedures for the development of the methodology discussed in the project proposal can only be outlined in their main features. This appendix provides such an outline.

I. Construction of all logically possible deterrence situations with:

A. Two principal actors, but allowing sub-actors or clients.

B. Thirteen possible preference orderings of basic outcomes (see Table 4).

C. Three general types of utility functions (RAV, RNE, RAC).

D. Two types of probability estimates (Pr(V), R(L)).

E. Three levels of deterrence (strategic war, limited war, potential crises).

These possibilities yield, before redundancies and trivial situations are eliminated, 9828 logically possible deterrence situations.

II. After these 9828 situations are constructed, a computer simulation will then be conducted,aimed at partitioning these situations into four subsets:

A. All trivial and/or redundant situations.

B. All situations in which deterrence always succeeds.

C. All situations in which deterrence never succeeds.

D. All situations in which deterrence depends on a decision maker's benefit-cost calculations and probability estimates.

III. Incorporation of dynamic considerations into the various deterrence situations. Here, two aspects are especially relevant: (1) variations in a decision maker's estimate of Pr(V) or R(L) over time; and (2) as we have used the "0,1-normalization" in defining utility functions, variations in the utility of a decision maker's middle ranked outcome (MRO), which in most relevant cases is SQ.

A. For Pr(V) or R(L), there are two cases and three possibilities in each case:

1. For specific values, Pr(V), suppose we consider two points in time t_o, t_1 and $t_1 > t_o$, then
   a. $Pr_v(t_1)=Pr_v(t_o)$ (This is the static case to be initially investigated.)
   b. $Pr_v(t_1) > Pr_v(t_o)$ (an increasing function.)
   c. $Pr_v(t_1) < Pr_v(t_o)$ (a decreasing function.)

2. For a range of values, R(L), in some interval [a;b], and points in time t_o, t_1 and $t_1 > t_o$, then
   a. [a;b] at t_o = [a';b'] at t_1, so R(L)=R(L)' (This is the static case to be initially investigated.)
   b. [a;b] at t_o > [a';b'] at t_1, so R(L)< R(L)' (an *increasing* function.)
   c. [a;b] at t_o < [a';b'] at t_1, so R(L)> R(L)'(a decreasing function.)

B. Using the same t_o, t_1 as above, there are three possibilities for the MRO:

1. $U_{MRO}(t_1)=U_{MRO}(t_o)$ (This is the static case to be initially investigated.)
2. $U_{MRO}(t_1) > U_{MRO}(t_o)$ (an increasing function.)
3. $U_{MRO}(t_1) < U_{MRO}(t_o)$ (a decreasing function.)

Specification of these dynamic situations adds an even greater number of deterrence situations to those already investigated. For example, for one actor, there are nine possible combinations of $Pr_v(t)$ or R(L) and $U_{MRO}(t)$, namely

$Pr_v(t)$ or $R(L)$	$U_{MRO}(t)$
static	static
static	increasing
static	decreasing
increasing	static
increasing	increasing
increasing	decreasing
decreasing	static
decreasing	increasing
decreasing	decreasing

For two actors, there would be 81 possible combinations. We will then repeat the earlier computer simulation, postulating various functional forms for $Pr_v(t)$, $R(L)$ and $U_{MRO}(t)$, e.g. $Pr_v(t)=1/t$, a monotonically decreasing function of t.

IV. Empirical application of the computer simulation in two stages: first, to the static deterrence situations: second, to the dynamic deterrence situations. We will follow these steps:

A. Selection for detailed study of a number of past deterrence situations (successes and failures.)
B. Gathering information relevant to the following questions:

   1. Who were the principal actors and sub-actors?
   2. At which level of deterrence was the specific situation located?
   3. As viewed by the principal actors, what were the possible outcomes of the deterrence situation?
   4. What were the preference orderings of each actor for the outcomes and how were their orderings constructed - that is, what factors were relevant to each actor's

benefit-cost calculations and how were these calculations carried out?

5. What possible courses of action were open to the principal actors?
6. If some action or actions were initially ruled out by the principal actors, why?
7. From the point of view of each principal actor, what outcome(s) was(were) assessed as likely to occur upon selection of each action?
8. Contingent upon his own actions, what responses did each principal actor believe possible for the other?
9. Were the various probability estimates made by the principal actors specific numbers or ranges of values?
10. Were the principal actors risk averse, neutral or acceptant?
11. How did the principal actors' perceptions of the situation change over time - that is, how did the answers to earlier questions vary, if at all?
12. What action did each principal actor select and why?
13. What was the eventual outcome of the deterrence situation?

C. Based on the hypothetical situations examined in the computer simulation and using the information gathered on the specific real world situations, we will make a "prediction" of each principal actor's course of action and the outcome of the specific deterrence situation and compare our "predictions" with what actually happened.

D. Analyze in detail any "false" predictions.

V. Using the knowledge and experience gained in the earlier stages of the project, we will select some present and future potential world trouble spots or likely crisis areas, e.g. Middle East, southern Africa and, as far as possible, replicate IV.

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